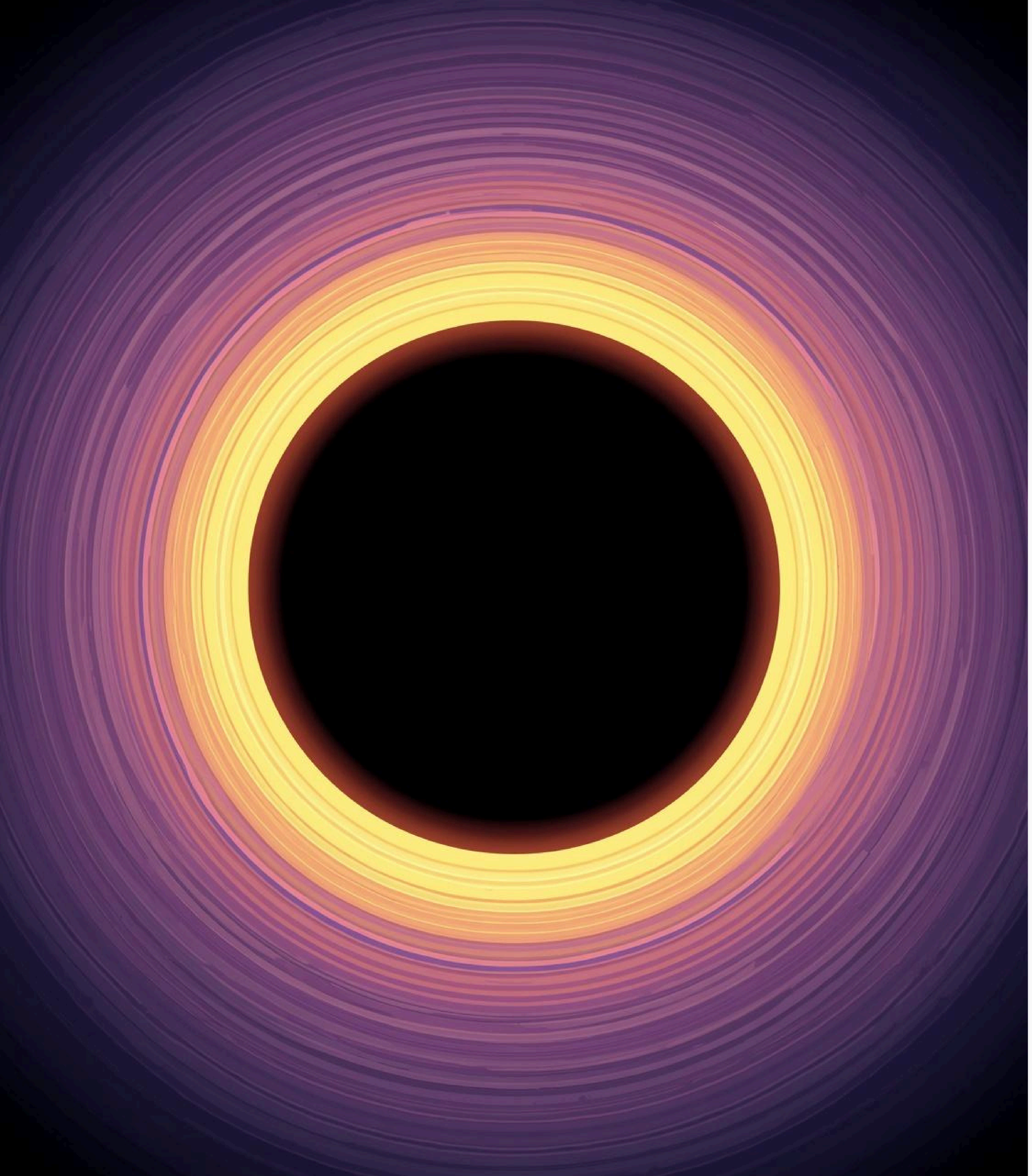

Generalized second law of thermodynamics in black-hole physics*1974

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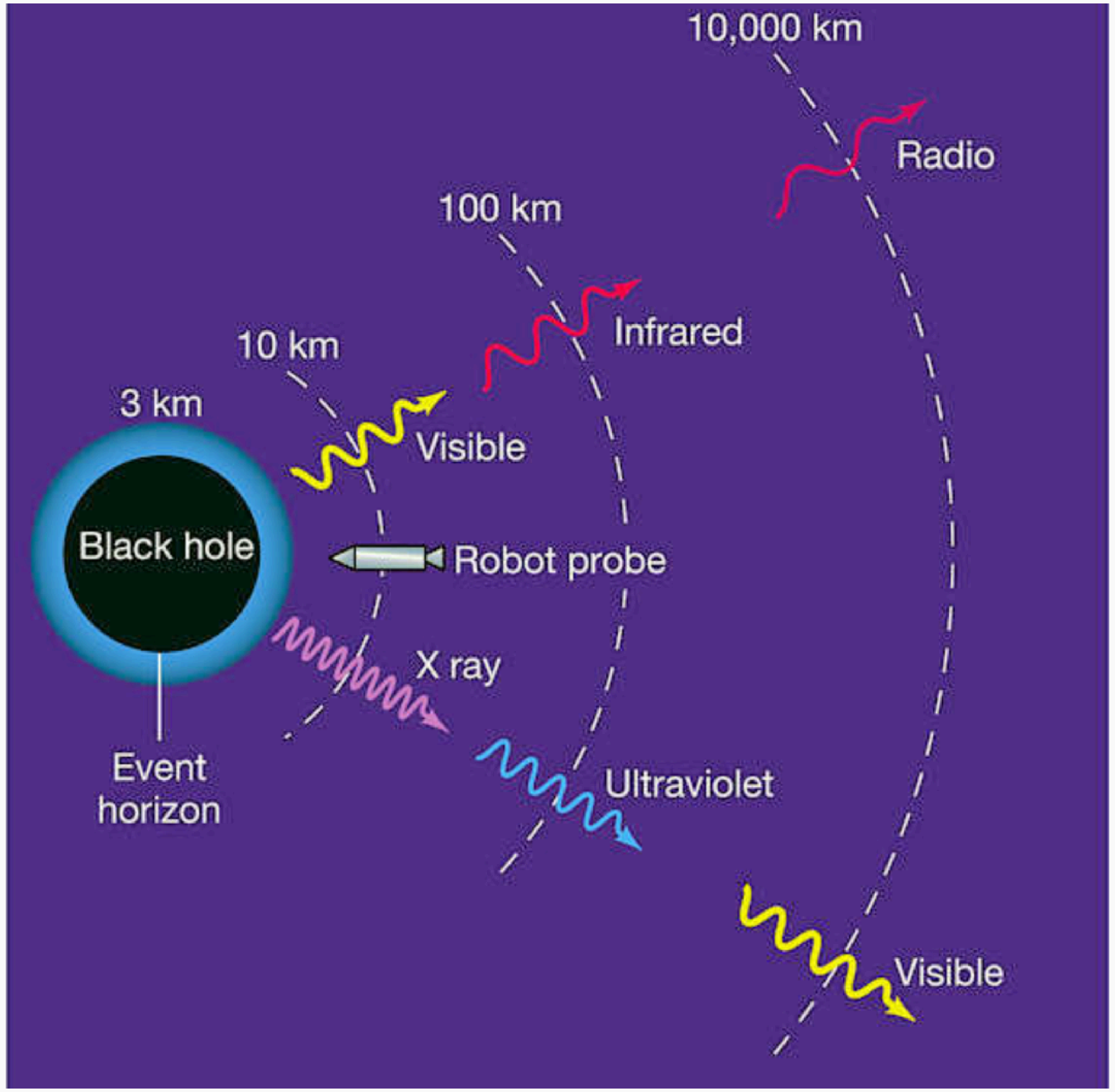
The Redshift Problem
Geroch's paradox

$$E_\infty = \sqrt{1 - \frac{2M}{r}}\, E_{\text{local}}$$

$$r \to 2M \quad \Longrightarrow \quad E_\infty \to 0$$

$$S_{\text{system}} > 0$$

$$\Delta S_{\text{gen}} < 0\ ?$$

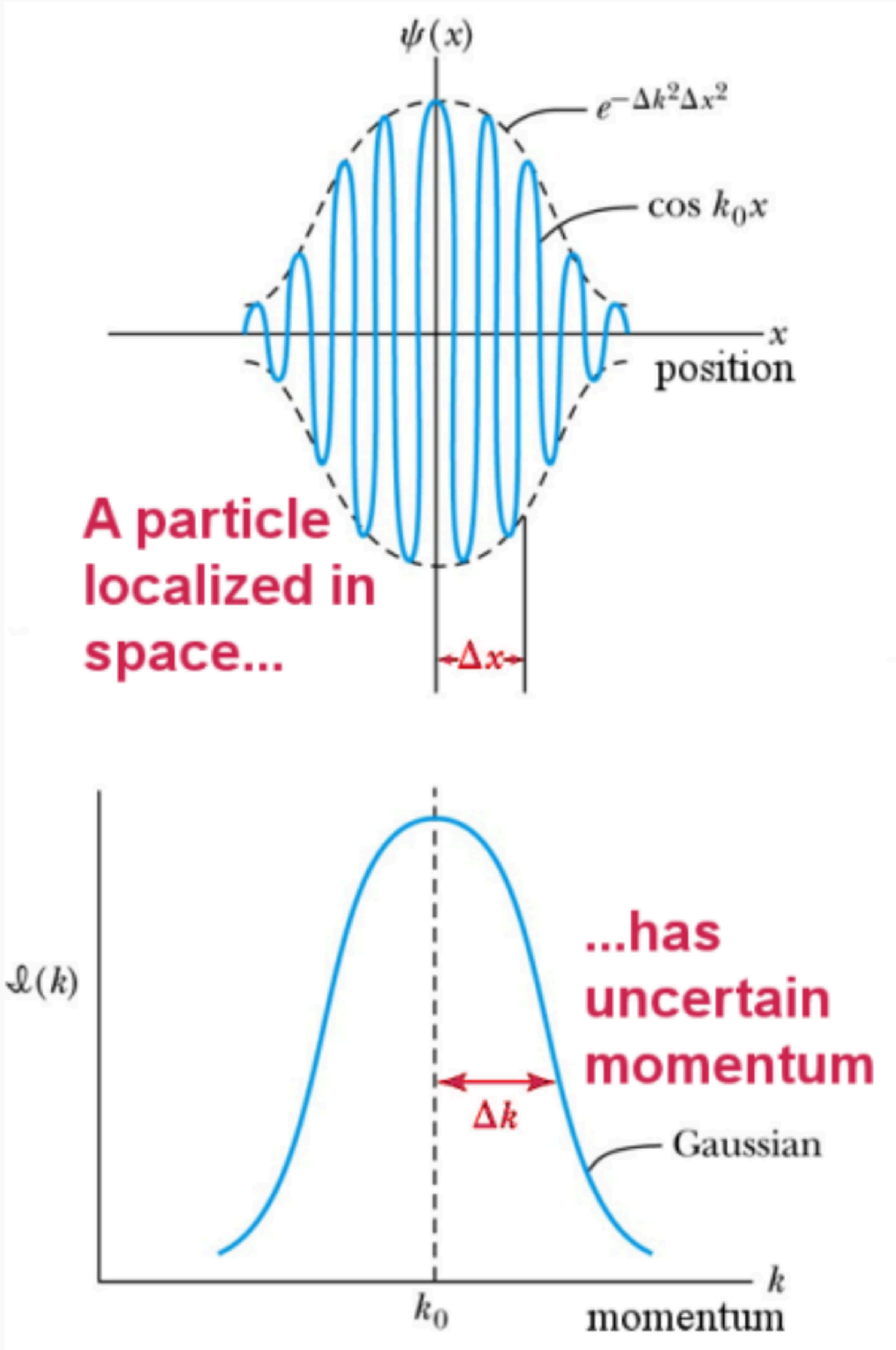


Quantum Limitation

$$\Delta x \gtrsim b$$

$$E_\infty \not\to 0$$

$$\Rightarrow \Delta S_{BH} > 0$$



Strategy of the Proof

$$\Delta S_{\mathrm{gen}} = \Delta S_{BH} + \Delta S_{\mathrm{outside}}$$

- Find **minimum** ΔS_{BH}
- Find **maximum** entropy loss outside
- Compare the two

Black Hole Entropy Increase

$$S_{bh} = (\tfrac{1}{2}\ln 2)\,\hbar^{-1}\alpha\,.$$

S_{bh}: black-hole entropy.
α: rescaled horizon area.
δA: infinitesimal horizon area element.
v: parameter along the horizon generators.
ρ: convergence of the null generators.
σ: shear of the null congruence.
$T_{\beta\gamma}$: stress-energy tensor.
l^{β}: null generator tangent vector.
T_{00}: local energy density.
x: distance from the horizon.
dV: local volume element.
μ: mass of the system.
b: size of the system.

$$\frac{d(\delta A)}{dv} = -2\rho\,\delta A$$

$$\frac{d\rho}{dv} = \rho^2 + |\sigma|^2 + 4\pi T_{\beta\gamma} l^{\beta} l^{\gamma}$$

$$\Longrightarrow \quad \frac{dA}{dv} = 2\int_v^{\infty} dv' \int_H \left(4\pi T_{\beta\gamma} l^{\beta} l^{\gamma} + |\sigma|^2 - \rho^2\right)\delta A(v')$$

$$\Longrightarrow \quad \Delta\alpha \geq 2\int_0^{\infty} dv \int_v^{\infty} dv' \int_H T_{\beta\gamma} l^{\beta} l^{\gamma}\,\delta A(v')$$

$$l^{\nu} = K(\lambda_0^{\nu} + n^i \lambda_i^{\nu}), \quad T_{\beta\gamma} l^{\beta} l^{\gamma} = K^2 (T_{00} + 2T_{0i} n^i + T_{ij} n^i n^j)$$

$$\Longrightarrow \quad \Delta\alpha \geq 2\int_V x\, T_{00}\, dV$$

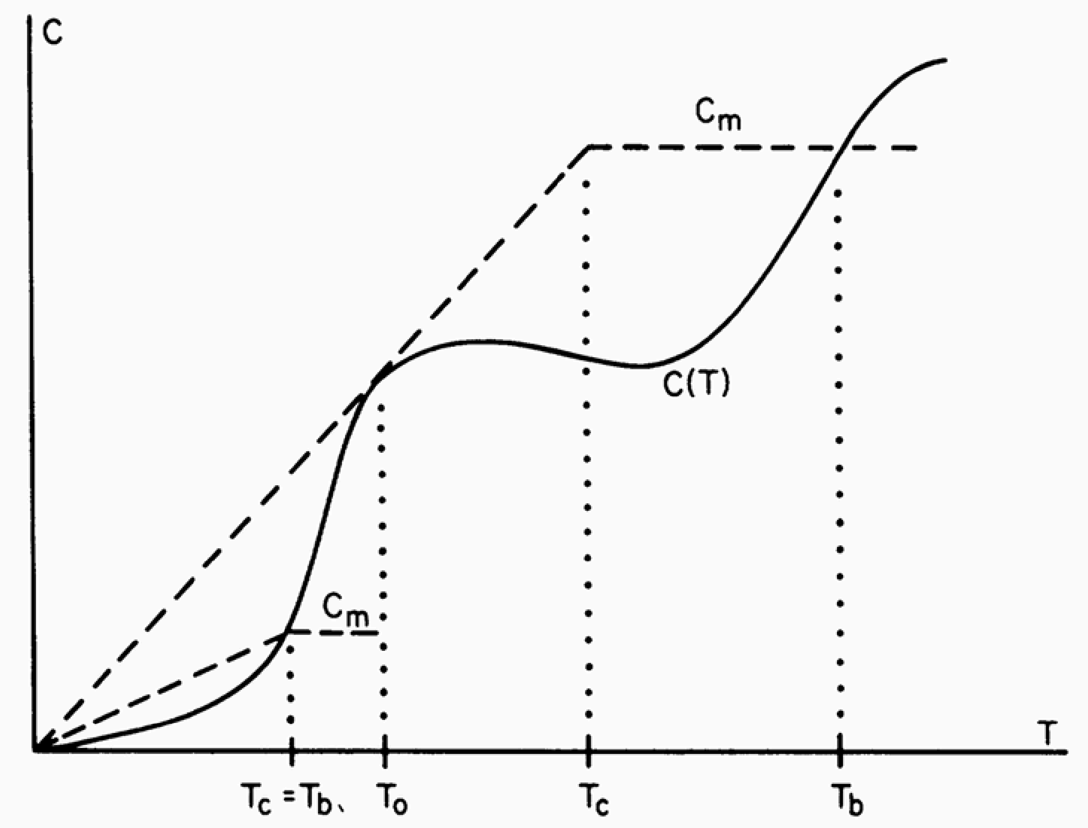
$$\Longrightarrow \quad \Delta\alpha \geq 2\mu b$$

$$\Longrightarrow \quad \Delta S_{bh} \geq \mu b\,\hbar^{-1} \ln 2.$$

Entropy of the Infalling System

$$T_b = \hbar (b \ln 2)^{-1}$$

$$-\Delta S_{\text{outside}} = S_{\text{system}}$$



$$T_b = \frac{\hbar}{b \ln 2}$$

$$S = S_0 + \int_0^T C(T') \, T'^{-1} \, dT' \, ,$$

$$\Delta S_{bh} \geq \frac{\mu}{T_b}$$

$$\Delta S_g \geq \frac{\mu}{T_b} - S$$

$$\Delta S_g \geq \frac{E_0}{T_b} - S_0 + \int_0^T C(T') \left(T_b^{-1} - T'^{-1} \right) dT'$$

Bekenstein bound

Universal upper bound on the entropy-to-energy ratio for bounded systems

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Phys. Rev. D **23**, 287 – **Published 15 January, 1981**

DOI: https://doi.org/10.1103/PhysRevD.23.287

$$S \leq \frac{2\pi E R}{\hbar c}$$

Generalized Second Law

$$\Delta S_{BH} \geq \Delta S_{\text{outside}}$$

$$\Rightarrow \Delta S_{\text{gen}} \geq 0$$

Impact on Modern Physics

holographic principle

The second law asserts that, in any classical physical process, the surface area of the event horizon cannot decrease, i.e., $\delta A \geq 0$. This mirrors the second law of thermodynamics, which states that the entropy of a closed system does not decrease.

- Entropy $\propto$ Area
- Information loss problem
- Hawking radiation
- Information paradox

$$S_{\text{gen}} = \frac{A}{4G\hbar} + S_{\text{out}}$$

$$S(\rho\|\sigma) = \Delta\langle K\rangle - \Delta S_{\text{out}}$$

Monotonicity of relative entropy: $S_{\text{rel}}(\Sigma_1) \geq S_{\text{rel}}(\Sigma_2)$

$$\Delta\langle K\rangle \leftrightarrow -\frac{\Delta A}{4G\hbar} \quad \text{(via semiclassical Einstein + Raychaudhuri)}$$

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_{ab}\sigma^{ab} - R_{ab}k^a k^b$$

$$\Longrightarrow \qquad S_{\text{gen}}(\Sigma_2) \geq S_{\text{gen}}(\Sigma_1)$$